2008
HIGHER SCHOOL CERTIFICATE
TRIAL EXAMINATION

Mathematics

General Instructions

- Reading time – 5 minutes
- Working time – 3 hours
- Write using black or blue pen
- Board-approved calculators may be used
- A table of standard integrals is provided at the back of this paper
- All necessary working should be shown in every question
- Start a new booklet for each question

Total marks – 120

- Attempt Questions 1 – 10
- All questions are of equal value

AD/BW 4th August 2008

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Question 1 (12 marks) Use a SEPARATE writing booklet **Marks**

(a) A telephone directory is 4.5 cm thick. There are 2000 pages in it. Find the thickness in millimeters of one page in scientific notation correct to 2 significant figures. **2**

(b) Factorise fully $8x^3 - 27$ **2**

(c) Solve for x: $|3x+1| = x+9$ **2**

(d) Find the gradient of the function $y = 3x^2 - 4x + 1$ at $(3,1)$ **2**

(e) Find the length of an arc of a circle with radius 15 cm if the arc subtends an angle of $70°$ at the centre (leave answer in terms of π) **2**

(f) Find the value of the pronumeral, giving reasons for your answer. **2**

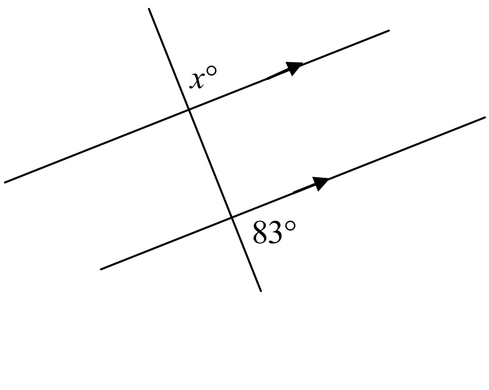
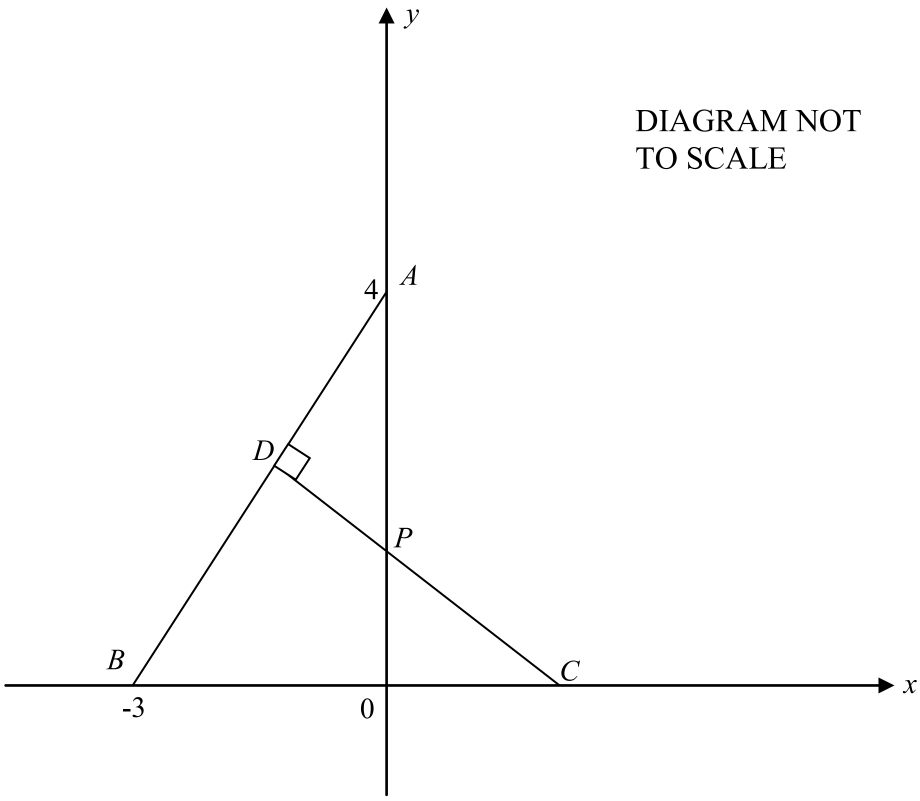


DIAGRAM NOT TO SCALE

End of Question 1

Question 2 (12 marks) Use a SEPARATE writing booklet. **Marks**



In the diagram $AB = BC$ and *CD* is perpendicular to *AB*.
CD intersects the *y* axis at *P*.

Copy the diagram onto your answer sheet.

(a) Find the length of *AB*. **1**

(b) Hence show the coordinates of *C* are $(2, 0)$ **1**

(c) Show the equation of *CD* is $3x + 4y = 6$ **3**

(d) Show the coordinates of *P* are $\left(0, 1\frac{1}{2}\right)$ **1**

(e) Use Pythagoras' Theorem on ΔPOC to show the length of *CP* is 2 ½ units. **1**

(f) Prove that ΔADP is congruent to ΔCOP **3**

(g) Hence calculate the area of the quadrilateral *DPOB* **2**

End of Question 2

Question 3 (12 marks) Use a SEPARATE writing booklet. **Marks**

(a) Differentiate with respect to x:

(i) $y = 3e^{4x} + x^2 - 1$ **1**

(ii) $y = \dfrac{3x-1}{2x+3}$ **2**

(b) (i) Find $\displaystyle\int \frac{3x}{x^2-4}\,dx$ **2**

(ii) Evaluate $\displaystyle\int_{\frac{\pi}{6}}^{\frac{\pi}{4}} 3\sin 2x \; dx$ **2**

(c) Using an appropriate substitution, solve for x: **3**

$$4^x - 12(2^x) + 32 = 0$$

(d) Prove $\tan\theta(1-\cot^2\theta) + \cot\theta(1-\tan^2\theta) = 0$ **2**

End of Question 3

Question 4 (12 marks) Use a SEPARATE writing booklet. **Marks**

(a) Graph the intersection of the regions $y < -x^2 + 3$ and $y \geq |x-3|$ **3**

(b) Let α and β be the roots of the equation $x^2 - 5x + 2 = 0$. Find the values of

(i) $\alpha + \beta$ **1**

(ii) $(\alpha + 1)(\beta + 1)$ **2**

(c) Find the values of k for which the quadratic equation $2x^2 - 4x + k = 0$ has one root. **2**

(d) Find the value of $\log_5 25$ **1**

(e) Given that $\frac{d^2s}{dt^2} = 10 - 2t,\ t \geq 0$, and $\frac{ds}{dt} = 24$ when $t = 0$, find t when $\frac{ds}{dt} = 0$. **3**

End of Question 4

Question 5 (12 marks) Use a SEPARATE writing booklet. **Marks**

(a) Alison decides to swim to improve her fitness levels.

On the first day she swims 450 metres, and on each day after that she swims 200 metres more than the previous day. That is she swims 650 metres on the second day, 850 metres on the third day and so on.

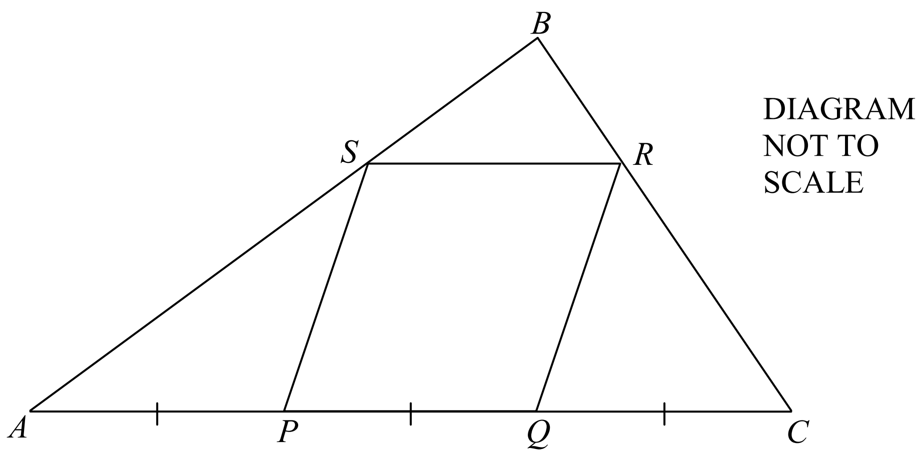
(i) How far does Alison swim on the 10^{th} day ? **1**

(ii) What is the total distance she swims in the first 10 days? **1**

(iii) Alison wants to enter an ocean swimming competition which is a distance of 5.25 kilometres. On which day will she be ready to complete this distance? **2**

(b) Find for x: $\log_2 x + \log_2 5 = 6$ **2**

(c)



The diagram above shows ΔABC, where $AP = PQ = QC$ and $PQRS$ is a rhombus.

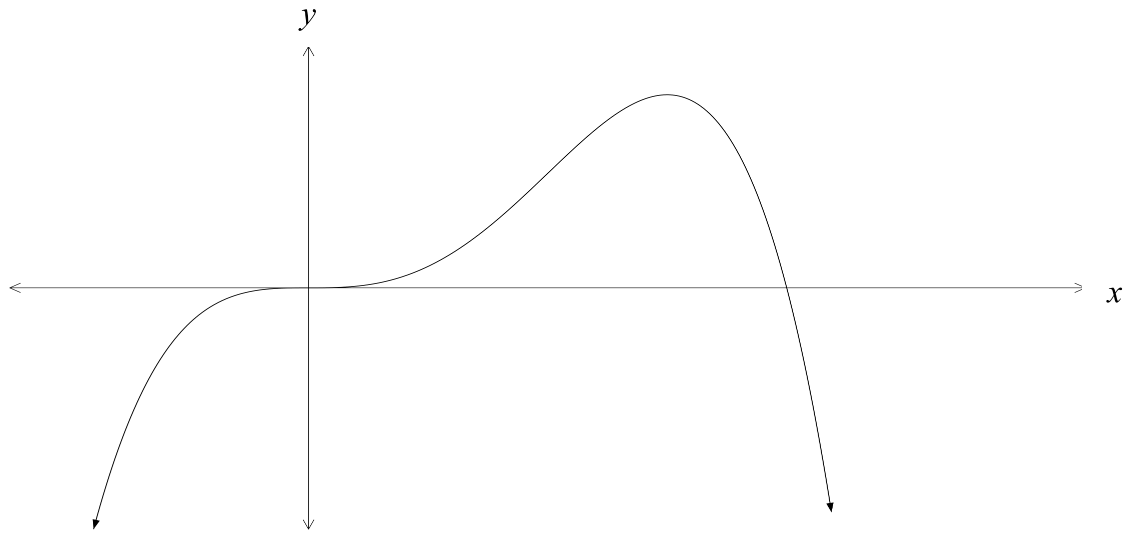
(i) If $\angle SAP = x°$ prove that $\angle SPQ = 2x°$ **2**

(ii) Prove that $\angle ABC = 90°$ **2**

Question 5 continued on page 8

Question 5 continued **Marks**

(d) Below is the graph of the function $y = f(x)$. In your answer booklet , draw a **2**
possible sketch of $y = f'(x)$



End of Question 5

Question 6 (12 marks) Use a SEPARATE writing booklet. **Marks**

(a) Express with a rational denominator $\dfrac{1}{3-\sqrt{2}}$ **2**

(b) (i) Express $0.1\dot{}$ as an infinite series. **1**

(ii) Hence express $0.1\dot{}$ as a fraction with no common factors. **1**

(c) The diagram shows a cross-section of a river at a point where the width *AB*, across the river, is 60 metres.

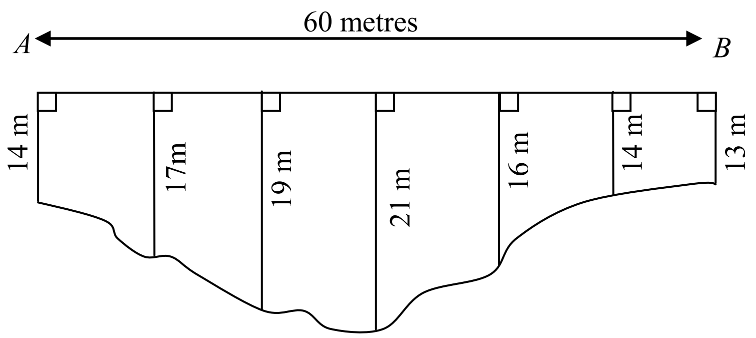


DIAGRAM NOT TO SCALE

(i) Using the Trapezoidal Rule and 6 strips, find the approximate area of the cross-section. **2**

(ii) At the point of the cross-section, the river is flowing at the rate of 80 cm/second. Calculate the volume of water that flows past this point in 1 second. **2**

(d) Find the equation of the normal to curve $y = \sin 2x \cos 2x$ at the point $(\pi, 0)$. **4**

End of Question 6

Question 7 (12 marks) Use a SEPARATE writing booklet. **Marks**

(a) Consider the function $f(x) = x^3 - 12x$

(i) Show that the curve represents an odd function. **2**

(ii) Show that the function has two stationary points and determine their nature. **3**

(iii) Find the coordinates of the point of inflection . **1**

(iv) Hence sketch the curve showing all important features **1**

(v) What is the relative maximum in the domain $-3 \le x \le 5$ **1**

(b) The limiting sum of a geometric series is 27

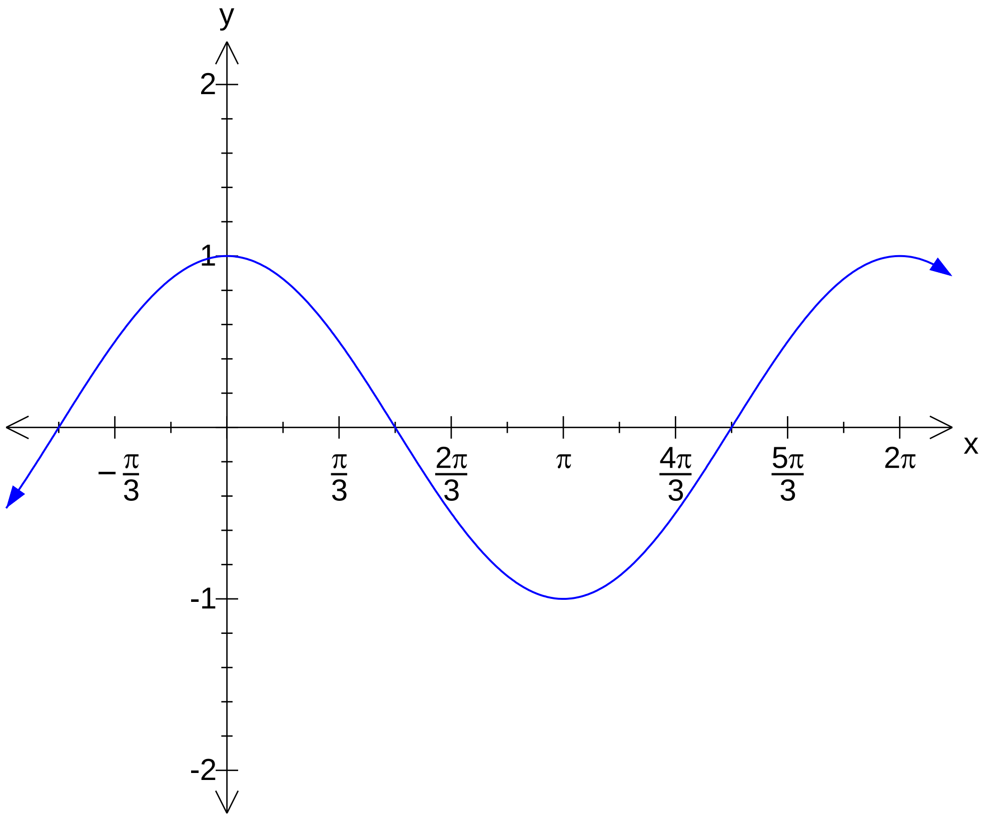
(i) Show that $a = 27(1-r)$. **1**

(ii) If the sum of the first three terms of the geometric series is 19, find the common ratio **3**

End of Question 7

Question 8 (12 marks) Use a SEPARATE writing booklet. **Marks**

(a) Below is the sketch of $y = \cos x$.
It is also provided for you on page 18 (Attachment A)



(i) Using attachment A, graph the function $y = \sqrt{3}\sin x$. **2**

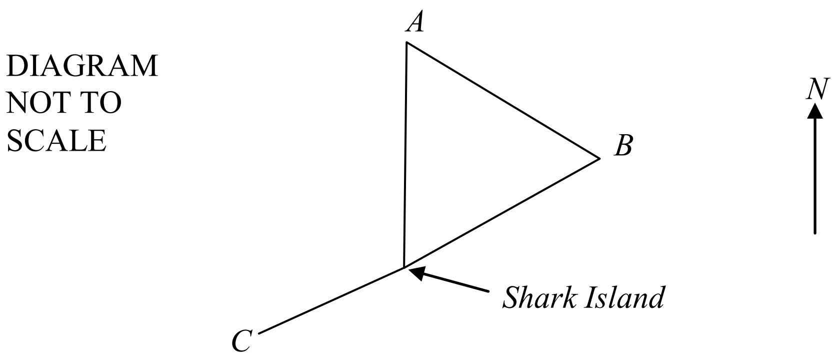
(ii) The curves intersect twice in the domain. Show, **algebraically**, that the x coordinates of the points of intersection are $x = \frac{\pi}{6}, \frac{7\pi}{6}$. **2**

(iii) Find the area between the two curves. **3**

Question 8 continued on page 12

Question 8 continued **Marks**

(b) Three boats A, B and C are situated off Shark Island, as shown below.



Boat A is due north of the island. Boat B is on a bearing of $060°$ from the island and Boat C is on a bearing of $240°$ from the island. Fish nets (of length 100 m) have been laid out between the island and each boat and also between boats A and B.

(i) Copy and complete the diagram showing all information. **1**

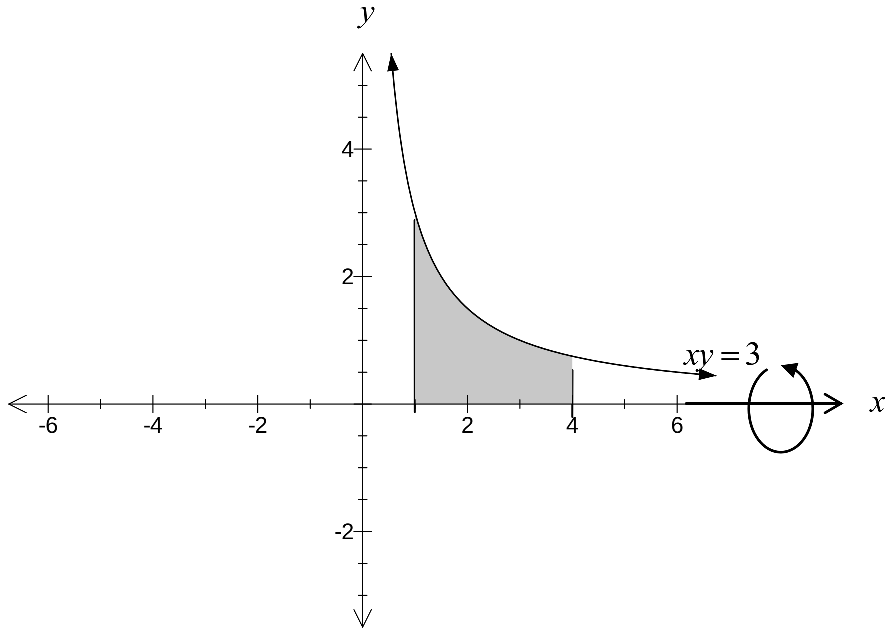
(ii) Calculate the triangular area that boats A and B have netted, to the nearest square metre. **2**

(iii) Calculate the length to the nearest metre of net needed between boats A and C. **2**

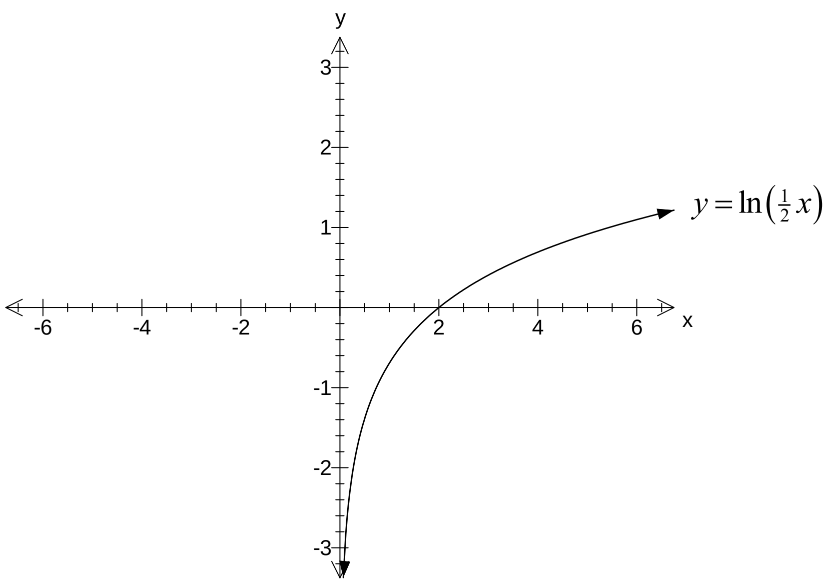
End of Question 8

Question 9 (12 marks) Use a SEPARATE writing booklet. **Marks**

(a) The area enclosed by the curve $xy = 3$, the x-axis and the lines $x = 1$ and $x = 4$, is rotated about the x-axis. Find the exact volume of the solid of revolution. **3**



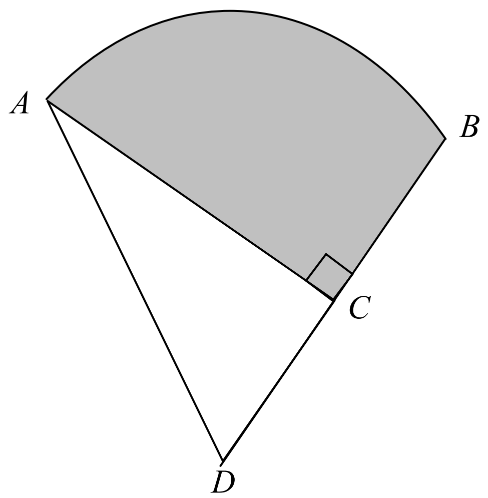
(b) The graph $y = \ln\left(\frac{1}{2}x\right)$ is shown below. Find the area between the x-axis, y – axis and the line $y = \ln 3$ **4**



Question 9 continued on page 14

Question 9 continued **Marks**

(c)



DIAGRAM
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ABD is a sector of a circle, centre *D*, such that $AD = BD = 24\ cm$, arc $AB = 4\pi\ cm$ and the line *AC* is perpendicular to *BD*.

(i) Show that $\angle ADB$ is $\frac{\pi}{6}$ **1**

(ii) Show that the exact length of DC is $12\sqrt{3}$ **1**

(iii) Find the exact area of ACB in terms of π **3**

End of Question 9

Question 10 (12 marks) Use a SEPARATE writing booklet. **Marks**

(a) Betina's daughter was born on the 1st August. On that day she opened a trust account by depositing \$500. Each year, on her birthday, she deposited \$500 into this trust account. She continued to do this up to and including her 17th birthday. When her daughter turned 18, Betina collected the total amount including interest from this account and presented it to her. This account paid interest at a rate of 6% per annum compounded every six months.

(i) Betina used the formula $A = P(1+r)^n$ to work out that her initial deposit amounted to $A = \$1449.14$ after 18 years. Write down values for P, r and n. **2**

(ii) After 2 years the amount in the account is A_2. **1**

$A_2 = 500(1.03^2 + 1.03^4)$

Write an expression for the amount in the account after 3 years, A_3.

(iii) Hence find the amount that Betina gives to her daughter on her 18th birthday. **3**

Question 10 continued on page 16

Question 10 continued **Marks**

(b) A circular stained glass window of radius $\sqrt{5}$ metres requires metal strips for support along *AB*, *DC* and *FG*.
O is the centre of the window.

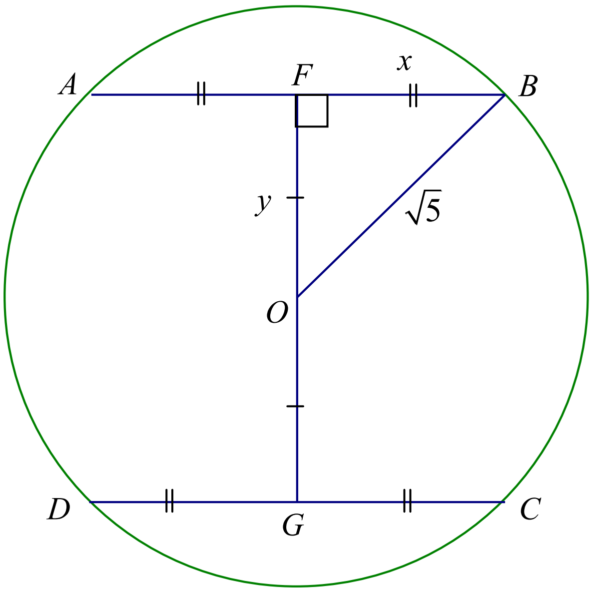

DIAGRAM
NOT TO
SCALE

(i) Copy the diagram and information onto your answer page.

(ii) If $OF = OG = y$ metres and $FB = x$ metres, find an expression for y in terms of x. **1**

(iii) Show that the total length L of the metal strips (i.e. $L = AB + DC + FG$) is given by: **1**

$$L = 4x + 2\sqrt{5 - x^2}\ .$$

(iv) The window will have maximum strength when the length of the supports is a maximum. Find the value of x that will allow the window to have maximum strength. **4**

End of Examination

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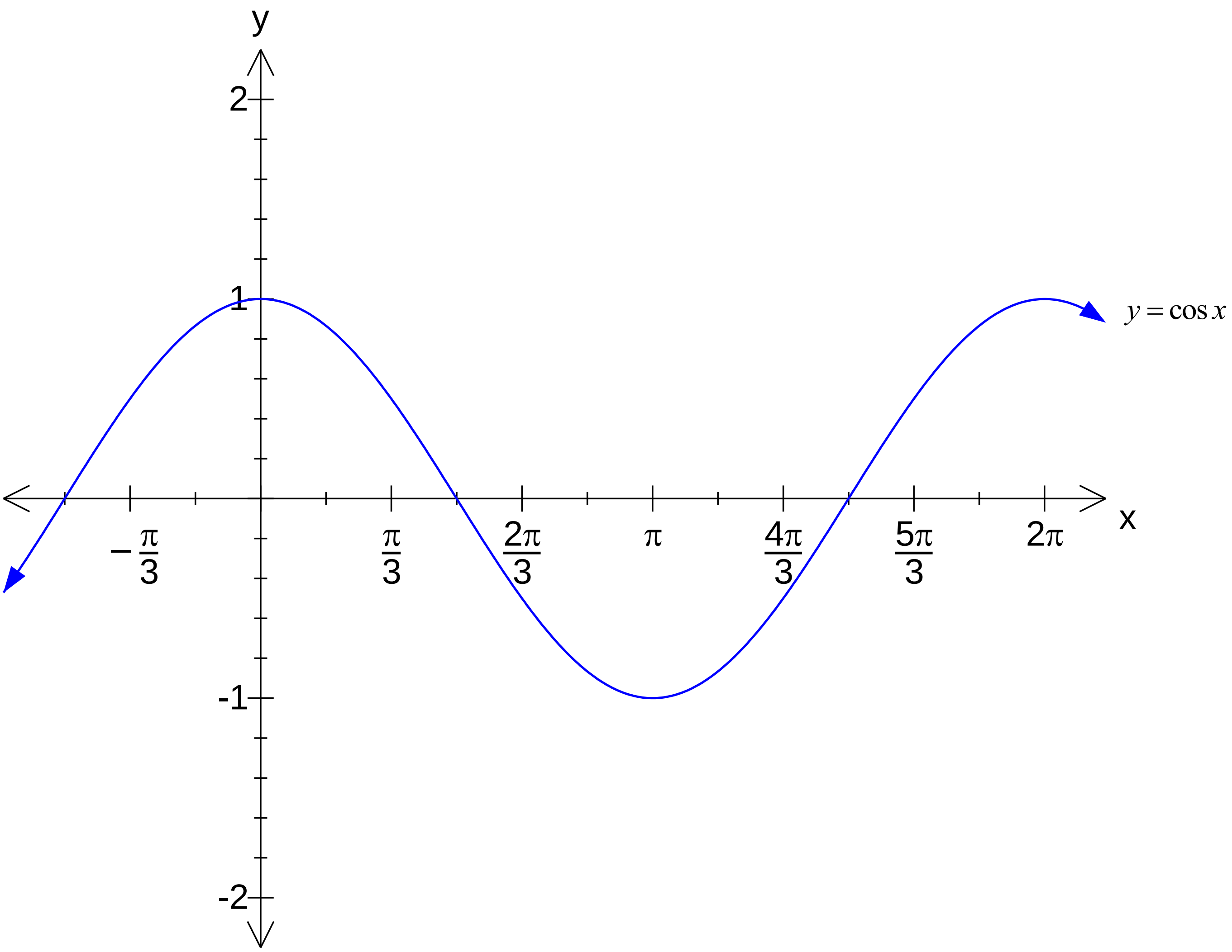
(ATTACHMENT A)

Question 8

(a) (i)



Solutions:

Question 1

(a) $4.5\ cm = 45mm$ ✓

$45 \div 2000 = 0.0225mm$

$2.25\times10^{-2}\ mm = 2.3\times10^{-2}\ mm\ \ (2\,sig\ figs)$ ✓

(b) $8x^3 - 27\ = (2x-3)(4x^2+6x+9)$ ✓✓

(c) $|3x+1| = x+9$

$3x+1 = x+9$ *or* $-3x-1 = x+9$ ✓

$2x = 8$ $\quad\quad -4x = 10$

$\therefore\ x = 4$ $\quad\quad x = -2.5$ ✓

(d) $y = 3x^2 - 4x + 1$

$\therefore \dfrac{dy}{dx} = 6x - 4$ ✓

$\therefore\ at\ x = 3\ \ \dfrac{dy}{dx} = 6(3) - 4 = 14$ ✓

(e) $70° = \dfrac{70\pi}{180} = \dfrac{7\pi}{18}\ radians$ ✓

since $l = r\theta$

$l = 15\times\dfrac{7\pi}{18}$

$l = \dfrac{35\pi}{6}\ \ cm$ ✓

(f) $x = 180 - 83$

$\therefore x = 97°$ ✓

(corresponding angles are equal on parallel lines)

(angles on a straight line are supplementary) ✓

Question 2

(a) $d_{AB} = \sqrt{(-3-0)^2 + (0-4)^2} = \sqrt{25}$

$= 5\ units$ ✓

(b) $C\ is\ 5\ units\ from\ B\ \therefore -3+5=2 \quad \therefore C(2,0)$ ✓

(c) $m_{AB} = \dfrac{4-0}{0--3} = \dfrac{4}{3}$ $\quad \sin ce\ \ AB \perp CD\ \ then\ m_{AB} = -\dfrac{3}{4}$ ✓

$\therefore\ y - 0 = -\dfrac{3}{4}(x-2)$ ✓

$4y = -3x + 6$

$\therefore\ 3x + 4y = 6$

(d) $P\ has\ coordinates\ (0, y)$

$\therefore\ 3x + 4y = 6$

$3(0) + 4y = 6$

$y = 1.5 \qquad \therefore P(0, 1.5)$ ✓

(e) $CP^2 = OC^2 + OP^2$

$CP^2 = 2^2 + 1.5^2$

$CP^2 = 4 + 2.25$

$CP^2 = 6.25$

$CP = \sqrt{6.25} = 2.5\ units$ ✓

(f) $\angle APD = \angle OPC$ $\quad (vertically\ opposite\ angles\ are\ equal)$ ✓

$AO = 4 units$

$OP = 1.5\ units$

$\therefore AP = AO - OP = 4 - 1.5 = 2.5$

$\therefore AP = CP = 2.5\ units$ ✓

$\angle ADP = \angle POC = 90°$ $\quad (given)$ ✓

$\therefore \Delta ADP \equiv \Delta\ COP\ \ (AAS)$

$(g)\quad Area_{\Delta COP} = \frac{1}{2}\times 2\times 1.5 = 1.5\ units^2$

$Area_{\Delta ABO} = \frac{1}{2}\times 3\times 4 = 6\ units^2$ ✓

$\therefore Area_{BDPO} = 6-1.5 = 4.5\ units^2$ ✓

Question 3

$(a)\ (i)\ \frac{dy}{dx} = 12e^{4x} + 2x$ ✓

$(ii)\ \frac{dy}{dx} = \frac{vu'-uv'}{v^2}$ $\qquad u = 3x-1 \quad u' = 3$

$v = 2x+3 \quad v' = 2$ ✓

$$\frac{dy}{dx} = \frac{vu'-uv'}{v^2} = \frac{3(2x+3)-2(3x-1)}{(2x+3)^2}$$

$$= \frac{11}{(2x+3)^2}$$ ✓

$(b)\ (i)\ \int \frac{3x}{x^2-4}\,dx = \frac{3}{2}\ln(x^2-4)+C$ ✓ ✓

$(ii)\ \int_{\frac{\pi}{6}}^{\frac{\pi}{4}} 3\sin 2x\ dx = \left[\frac{-3\cos 2x}{2}\right]_{\frac{\pi}{6}}^{\frac{\pi}{4}}$ ✓

$$= \left[\frac{-3\cos 2\left(\frac{\pi}{4}\right)}{2} - \frac{-3\cos 2\left(\frac{\pi}{6}\right)}{2}\right]$$

$$= \left[0 + \frac{3\times\frac{1}{2}}{2}\right]$$

$$= \frac{3}{4}$$ ✓

(c) $let\ u = 2^x$

$$u^2 - 12u + 32 = 0 \quad ☑$$

$$\therefore (u-8)(u-4) = 0$$

$$2^x = 8 \quad and \quad 2^x = 4 \quad ☑$$

$$\therefore\ x = 3 \qquad x = 2 \quad ☑$$

(d) $\tan\theta(1-\cot^2\theta) + \cot\theta(1-\tan^2\theta) = 0$

$LHS:\quad \tan\theta - \tan\theta\cot^2\theta + \cot\theta - \cot\theta\tan^2\theta$

$$= \frac{\sin\theta}{\cos\theta} - \frac{\sin\theta}{\cos\theta}\times\frac{\cos^2\theta}{\sin^2\theta} + \frac{\cos\theta}{\sin\theta} - \frac{\cos\theta}{\sin\theta}\times\frac{\sin^2\theta}{\cos^2\theta} \quad ☑$$

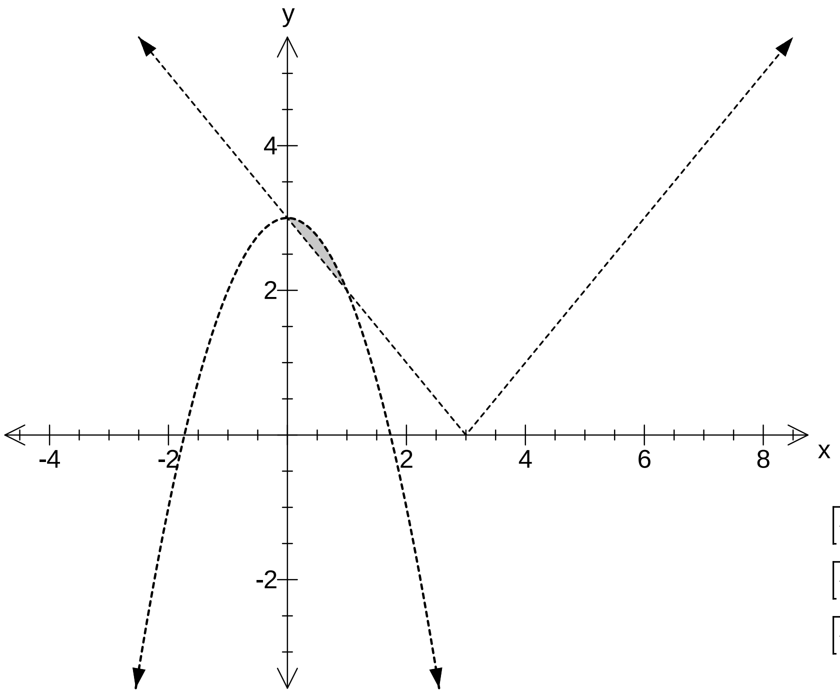
$$= \frac{\sin\theta}{\cos\theta} - \frac{\cos\theta}{\sin\theta} + \frac{\cos\theta}{\sin\theta} - \frac{\sin\theta}{\cos\theta} \quad ☑$$

$$= 0$$

$$= RHS$$

Question 4

(a)



☑ $graph\ y < -x^2 + 3$

☑ $graph\ y \geq |x-3|$

☑ rsection

(b) (i) $\alpha+\beta=\frac{-b}{a}=\frac{--5}{1}=5$ ✓

(ii) $(\alpha+1)(\beta+1)=\alpha\beta+\alpha+\beta+1$

$=2+5+1$ ✓

$=8$ ✓

(c) $2x^2-4x+k=0$ $\quad \therefore \Delta=0$ ✓

$\therefore \Delta=b^2-4ac$

$=(-4)^2-4(2)(k)$

$=16-8k=0$

$\therefore\ k=2$ ✓

(d) $\frac{\log_e 25}{\log_e 5}=2$ ✓

(e) $\frac{d^2s}{dt^2}=10-2t$

$\frac{ds}{dt}=10t-t^2+C \quad \sin ce\ \frac{ds}{dt}=24\ when\ t=0 \quad \therefore C=24$ ✓

$\frac{ds}{dt}=10t-t^2+24=0$

$-(t^2-10t-24)=0$

$-(t-12)(t+2)=0$

$\therefore t=12\ \ or\ \ t=-2$ ✓

since $t\geq 0 \quad then\ t=12$ ✓

<u>Question 5</u>

(a) (i) $AP:\ T_n = a + (n-1)d$

$$T_{10} = 450 + (10-1)\times 200$$

$$= 2250\ m \quad \checkmark$$

(ii) $AP:\ S_n = \frac{n}{2}(2a + (n-1)d)$

$$S_{10} = \frac{10}{2}(2(450) + (10-1)200)$$

$$= 13500\,m \quad \checkmark$$

(iii) $5.25\ kilometres = 5250\,m$

$$T_n = a + (n-1)d$$

$$5250 = 450 + (n-1)\times 200 \quad \checkmark$$

$$\therefore n = 25th\ day \quad \checkmark$$

(b) $\log_2 x + \log_2 5 = 6$

$$\therefore \log_2 5x = 6 \quad \checkmark$$

$$5x = 2^6$$

$$x = 12.8 \quad \checkmark$$

(c) (i) since $PQRS$ is a rhombus then $PQ = AP = SP$ $\quad \therefore \Delta APS\ is\ isosceles$ ✓

$if\ \ \angle SAP = x^\circ\ \ then\ \angle ASP = x^\circ$

$\therefore \angle SPQ = 2x\ \ (exterior\ angle\ theorem\ of\ a\ triangle)$ ✓

(ii) $\angle SPQ = 2x^\circ$

$\angle\ RQP = 180 - 2x\ \ (co-interior\ angles\ as\ SP \parallel RQ)$

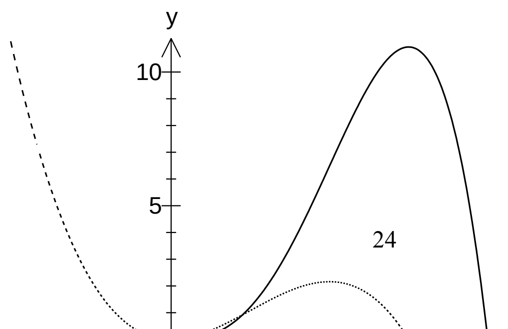
$\sin ce\ \ PQ = QR = QC\ \ then\ \ \Delta QRC\ is\ an\ isosceles\ \Delta$

$\therefore \angle QRC = \angle RCQ = 90 - x$ ✓

$\sin ce\ \ \angle SAP = x^\circ\ \ and\ \angle RCQ = 90 - x$

$then\ \ \Delta ABC = 180 - (90 - x) - x = 90^\circ\ \ \ (angle\ sum\ of\ a\ \Delta)$ ✓

(d)



☑ ative of the HPOI at $x = 0/2nd$ TP

☑ l x intercepts

Question 6

(a) $\frac{1}{3-\sqrt{2}} \times \frac{3+\sqrt{2}}{3+\sqrt{2}}$ ☑

$= \frac{3+\sqrt{2}}{9-4}$

$= \frac{3+\sqrt{2}}{7}$ ☑

(b) (i) $0.1\dot{}$ $\frac{1}{10} + \frac{5}{100} + \frac{5}{1000} + \frac{5}{10000} + \ldots$ ☑

(ii) $0.1\dot{}$ $\frac{1}{10} + \frac{5}{100} + \frac{5}{1000} + \frac{5}{10000} + \ldots$

$S_\infty = \frac{a}{1-r} = \frac{\frac{5}{100}}{1-\frac{1}{10}} = \frac{1}{18}$

$\therefore \frac{1}{10} + \frac{1}{18} = \frac{7}{45}$ ☑

(c) (i) $A \approx \frac{10}{2}\left(14+2(17+19+21+16+14)+13\right)$ ✓✓

$A \approx 1005\ m^2$

(ii) $V = A \times h$

$= 1005 \times 0.8 m/s$ ✓

$= 804\ m^3$ ✓

(d) $y = \sin 2x \cos 2x$

$u = \sin 2x \qquad v = \cos 2x$

$u' = 2\cos 2x \qquad v' = -2\sin 2x$

$\therefore \frac{dy}{dx} = vu' + uv'$

$= 2\cos^2 2x - 2\sin^2 2x$ ✓

$at\ x = \pi \quad \frac{dy}{dx} = 2\cos^2 2(\pi) - 2\sin^2 2(\pi)$

$= 2 - 0$

$= 2$ ✓

$\therefore m_T = 2$

$\therefore m_N = -\frac{1}{2} \quad at\ (\pi, 0)$ ✓

$y - 0 = -\frac{1}{2}(x - \pi)$

$2y = -x + \pi$ ✓

Question 7

(a) (i) $y = x^3 - 12x$

$f(x) = x^3 - 12x$

$f(-x) = (-x)^3 - 12(-x) = -x^3 + 12x$

$-f(x) = -(x^3 - 12x) = -x^3 + 12x$ ✓

$Since\ f(-x) = -f(x)\ then\ y = y = x^3 - 12x\ \ is\ an\ odd\ function$ ✓

(ii) $y = x^3 - 12x$

$\frac{dy}{dx} = 3x^2 - 12$

$SP\ occur\ when\ \frac{dy}{dx} = 0 \qquad \therefore \frac{dy}{dx} = 3x^2 - 12 = 0$

$3(x^2 - 4) = 0$

$3(x+2)(x-2) = 0 \qquad \therefore x = 2\ or\ -2$ ✓

$\frac{d^2y}{dy^2} = 6x$

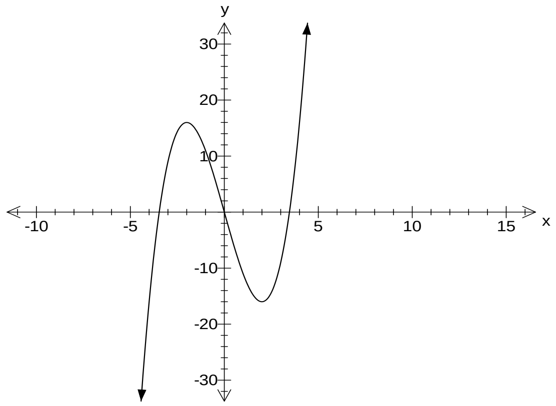
$at\ x = 2 \quad \frac{d^2y}{dy^2} = 6(2) = 12 \qquad \sin ce\ \frac{d^2y}{dy^2} > 0 \qquad \therefore \min\ at\ (2, -16)$ ✓

$at\ x = -2 \quad \frac{d^2y}{dy^2} = 6(-2) = -12 \qquad \sin ce\ \frac{d^2y}{dy^2} < 0 \qquad \therefore \max\ at\ (-2, 16)$ ✓

(iii) *Inflection occur when* $\dfrac{d^2y}{dy^2}=0$

$$\therefore \frac{d^2y}{dy^2}=6x=0 \qquad \therefore POI\ is\ (0,\ 0)$$

(iv)



✓

(v) $at\ x=-3\quad y=16$

$at\ x=5\quad y=65 \qquad \therefore relative\ \max imum\ is\ 65$ ✓

(b) (i) $S_\infty=\dfrac{a}{1-r}=27$

$$\therefore a=27(1-r)$$ ✓

(ii) $S_3=\dfrac{a(1-r^3)}{(1-r)}=19$ ✓

$$=\frac{27(1-r)(1-r^3)}{(1-r)}=19$$

$$27(1-r^3)=19$$ ✓

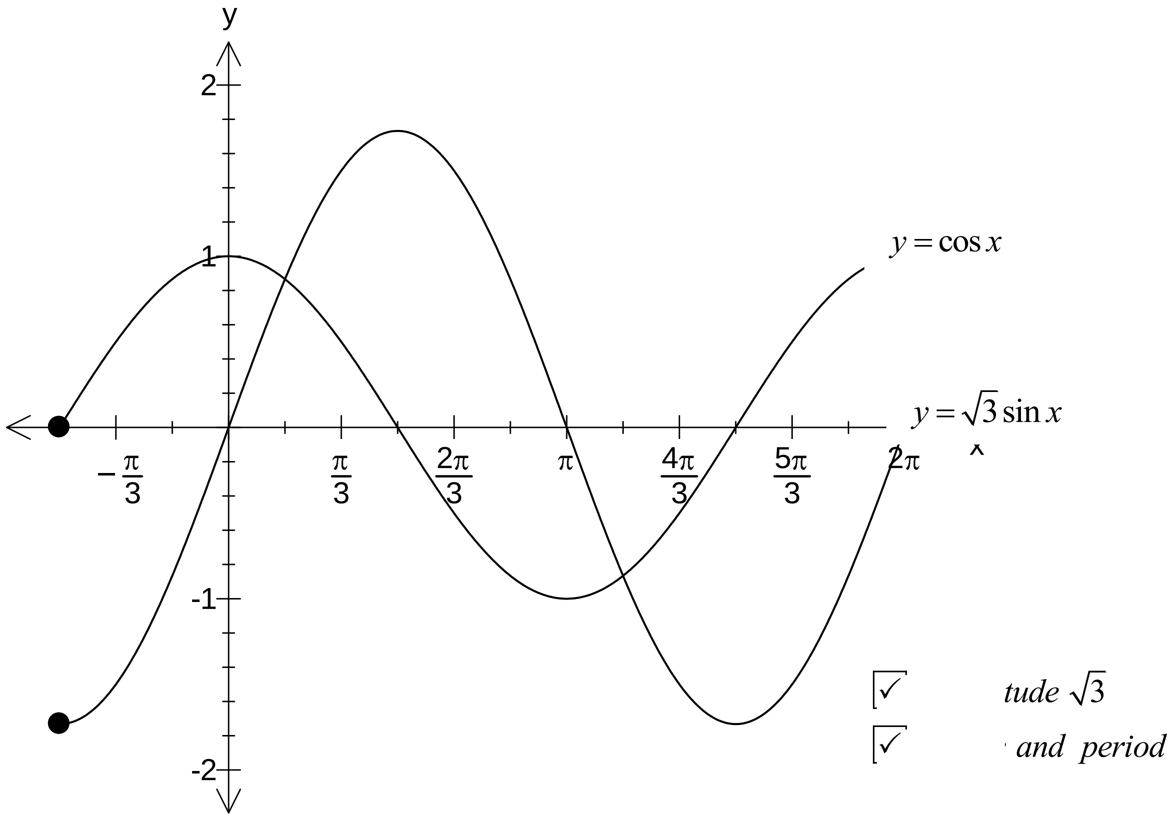
$$1-r^3=\frac{19}{27}$$

$$-r^3=\frac{-8}{27}$$

$$\therefore r=\frac{2}{3}$$ ✓

Question 8

(a)



(ii) $y=\sqrt{3}\sin\left(\frac{\pi}{6}\right)=\frac{\sqrt{3}}{2}$ $\quad y=\cos\left(\frac{\pi}{6}\right)=\frac{\sqrt{3}}{2}$ $\quad \therefore \frac{\pi}{6}$ *is a solution* ✓

$y=\sqrt{3}\sin\left(\frac{7\pi}{6}\right)=\frac{-\sqrt{3}}{2}$ $\quad y=\cos\left(\frac{7\pi}{6}\right)=\frac{-\sqrt{3}}{2}$ $\quad \therefore \frac{7\pi}{6}$ *is a solution* ✓

(iii) $\int_{\frac{\pi}{6}}^{\frac{7\pi}{6}} \sqrt{3}\sin x-\cos x\, dx$ ✓

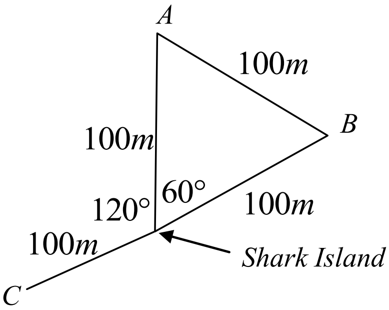
$$=\left[-\sqrt{3}\cos x-\sin x\right]_{\frac{\pi}{6}}^{\frac{7\pi}{6}} \quad ✓$$

$$=\left[-\sqrt{3}\cos\left(\frac{7\pi}{6}\right)-\sin\left(\frac{7\pi}{6}\right)\right]-\left[-\sqrt{3}\cos\left(\frac{\pi}{6}\right)-\sin\left(\frac{\pi}{6}\right)\right]$$

$$=\left(\frac{3}{2}+\frac{1}{2}\right)-\left(-\frac{3}{2}-\frac{1}{2}\right)$$

$$=4\ units^2 \quad ✓$$

(*b*) (*i*)



✓ m

(*ii*) $A = \frac{1}{2} \times 100 \times 100 \times \sin 60$ ✓

$= 4330\ m^2$ ✓

(*iii*) $AC^2 = 100^2 + 100^2 - 2 \times 100 \times 100 \times \cos(360 - 240)$ ✓

$AC^2 = 30000$

$AC = \sqrt{30000}$

$\therefore AC = 173\ m$ ✓

Question 9

(*a*) $V = \pi \int_1^4 \left(\frac{3}{x}\right)^2 dx$ ✓

$= \pi \left[\frac{-9}{x}\right]_1^4$ ✓

$= \pi \left[\frac{-9}{4} - \frac{-9}{1}\right]_1^4$

$= \frac{27\pi}{4}\ unit^3$ ✓

(*b*) $y = \log_e \frac{1}{2} x$

$\therefore e^y = \frac{1}{2} x$

$x = 2e^y$ ✓

$\int_0^{\ln 3} 2e^y\ dy$ ✓

$= \left[2e^y\right]_0^{\ln 3} = \left[2e^{\ln 3} - 2e^0\right] = 6 - 2 = 4\ units^2$ ✓

$(c)\ (i)\quad l = r\theta$

$4\pi = 24 \times \theta$

$\therefore \theta = \dfrac{4\pi}{24} = \dfrac{\pi}{6}$ ✓

$(ii)\quad \cos\theta = \dfrac{DC}{AD}$

$\cos\dfrac{\pi}{6} = \dfrac{DC}{24}$

$\therefore DC = \cos\dfrac{\pi}{6} \times 24 = \dfrac{\sqrt{3}}{2} \times 24 = 12\sqrt{3}$ ✓

$(iii)\quad A_{\sec tor\ ABD} = \dfrac{1}{2} \times 24^2 \times \dfrac{\pi}{6} = 48\pi$ ✓

$\sin\dfrac{\pi}{6} = \dfrac{AC}{24} \quad \Rightarrow \quad AC = 12$ ✓

$A_{\Delta ACD} = \dfrac{1}{2} \times 12 \times 12\sqrt{3} = 72\sqrt{3}$ ✓

$\therefore A_{ACB} = 48\pi - 72\sqrt{3} = 24(2\pi - 3\sqrt{3})cm^2$

Question 10

$(i)\quad 18 years = 36\ half\ \ years$

$6\%\, p.a = 3\%\ p.h$

$\therefore r = 3\% \qquad n = 36 \quad and \quad P = \$500$ ✓

$\therefore A_1 = 500(1 + 0.03)^{36} = \$1449.14$ ✓

$(ii)\quad A_2 = 500(1+0.03)^2 + 500(1+0.03)^4 = 500(1.03^2 + 1.03^4)$

$\therefore\ A_3 = 500(1.03^2 + 1.03^4 + 1.03^6)$ ✓

$(iii)\quad A_{18} = 500(1.03^2 + 1.03^4 + 1.03^6 + ... + 1.03^{36})$ ✓

$S_{36} = \dfrac{1.03^2((1.03^2)^{18} - 1)}{1.03^2 - 1} = 33.06869422$ ✓

$\therefore\ A_{18} = 500 \times 33.06869422 = \$16534.35$ ✓

(b) (ii) $\sqrt{5}^2 = x^2 + y^2 \quad \therefore y = \sqrt{5 - x^2}$ ✓

(iii)
$$\begin{aligned} L = AB + DC + FG &= 2x + 2x + 2y \\ &= 4x + 2y \\ &= 4x + 2\sqrt{5 - x^2} \end{aligned}$$ ✓

(iv)
$$\begin{aligned} L &= 4x + 2\left(5 - x^2\right)^{\frac{1}{2}} \\ \frac{dL}{dx} &= 4 + 2 \times \frac{1}{2} \times -2x\left(5 - x^2\right)^{-\frac{1}{2}} \\ &= 4 - \frac{2x}{\sqrt{5 - x^2}} \end{aligned}$$

max/ min *occur when* $\frac{dL}{dx} = 0 \quad \therefore\ 4 - \frac{2x}{\sqrt{5 - x^2}} = 0$ ✓

$$\frac{2x}{\sqrt{5 - x^2}} = 4 \qquad (square\ both\ sides)$$

$$4x^2 = 16(5 - x^2)$$

$$20x^2 = 80$$

$$x^2 = 4$$

$$x = \pm 2$$ ✓

$$\frac{dL}{dx} = 4 - 2x\left(5 - x^2\right)^{-\frac{1}{2}} \qquad \therefore \frac{d^2L}{dx^2} = 4 - (vu' + uv')$$

$$= 4 - \left(\left(\left(5 - x^2\right)^{-\frac{1}{2}} \times 2 + 2x \times \frac{-1}{2}\left(5 - x^2\right)^{-\frac{3}{2}} \times -2x\right)\right)$$

$$= 4 - \left(\left(\frac{2}{\sqrt{5 - x^2}} + \frac{2x^2}{\left(\sqrt{5 - x^2}\right)^3}\right)\right)$$ ✓

since x *is a length then* $x > 0 \quad \therefore test\ x = 2$

at $x = 2 \qquad \frac{d^2L}{dx^2} = 4 - \left(\left(\frac{2}{\sqrt{5 - 4}} + \frac{8}{\left(\sqrt{5 - 4}\right)^3}\right)\right) < 0 \ \therefore$ max ✓

$\therefore$ maximum strength in window frame when $x = 2\,metres$